

L'espace de Teichmüller : de la basse dimension à l'infini et au-delà

1-2 juin 2023
SC 10.01

Jeudi 1 juin		Vendredi 2 juin	
09:00 - 09:30	Welcome and coffee	09:30 - 10:30	Yilin Wang
09:30 - 10:30	Athanase Papadopoulos	10:30 - 11:00	Coffee break
10:45 - 11:45	Jean-Marc Schlenker	11:00 - 12:00	Dragomir Saric
11:45 - 13:30	Lunch	12:00 - 14:00	Lunch
13:30 - 14:30	Tudor Ratiu	14:00 - 15:00	Jeff Danciger
14:45 - 15:45	Alice Barbara Tumpach	15:00 - 15:30	Coffee break
15:45 - 16:30	Coffee break	15:30 - 16:30	Vlad Markovic
16:30 - 17:30	Andrea Seppi		
19:30 - 21:30	Dinner		

Jeff DANCIGER (Austin/IHES)

Eigenvalue asymmetry for convex real projective surfaces

A convex real projective surface is one obtained as the quotient of a properly convex open set in the projective plane by a discrete subgroup of $SL(3, \mathbb{R})$, called the holonomy group, that preserves this convex set. The most basic examples are hyperbolic surfaces, for which the convex set is bounded by an ellipse, and the holonomy group is conjugate into $SO(2, 1)$. In this case, the eigenvalues of elements of the holonomy group are symmetric. More generally, the asymmetry of the eigenvalues of the holonomy group is a natural measure of how far a convex real projective surface is from being hyperbolic. We study the problem of determining which elements (or generally geodesic currents) may have maximal eigenvalue asymmetry. We will present some limited initial results that we hope may be suggestive of a bigger picture. Joint work with Florian Stecker.

Vlad MARKOVIC (Oxford)

Unramified correspondance and virtual homology of mapping class groups

I shall discuss my recent work showing that the Bogomolov-Tschinkel universality conjecture holds if and only if the mapping class groups of a punctured surface is large (which is essentially the negation of the Ivanov conjecture about the mapping class groups).

Athanase PAPADOPOULOS (Strasbourg)

A survey of the Finsler geometry of Teichmüller space

will survey several Finsler metrics on Teichmüller space, pointing out relations among each other.

Tudor RATIU (EPFL/Shanghai)

Teichmüller space and reduction

Dragomir ŠARIĆ (Queens College)

Riemann surfaces of class O_G , the Fenchel-Nielsen parameters and holomorphic quadratic differentials

We consider infinite Riemann surfaces equipped with conformal hyperbolic metrics. The Fenchel-Nielsen parameters on geodesic pants decompositions completely recover hyperbolic metrics and we describe sufficient conditions on the Fenchel-Nielsen parameters to guarantee that the geodesic flow is ergodic. The ergodicity of the geodesic flow is known to be equivalent to the non-existence of Green's function-i.e. surfaces of class O_G . The results are joint work with Ara Basmajian and Hrant Hakobyan, and with Michael Pandazis.

We also characterize which measured foliations appear as horizontal foliations of finite-area holomorphic quadratic differentials on infinite surfaces (an extension of Hubbard-Masur theorem). As a corollary, we establish a new characterization of the class O_G and an extension of Kerckhoff's formula for the Teichmüller metric on the Teichmüller spaces of the O_G Riemann surfaces. Finally, we present a recent improvement in the detection whether certain choices of the Fenchel-Nielsen parameters give O_G surfaces in the case of surfaces with Cantor set of topological ends due to M. Pandazis. The condition is deduced using the above characterization of O_G surfaces in terms of finite-area holomorphic quadratic differentials.

Jean-Marc SCHLENKER (Luxembourg)

The renormalized volume of convex co-compact manifolds

Convex co-compact hyperbolic manifolds have infinite volume. However, they have a well-defined "renormalized" volume, which has a number of interesting properties. We will outline the definition of this renormalized volume, and some of its key properties, including its relations to the volume of the convex core. We will then survey some recent results on the global behavior of the renormalized volume, considered as a function over the deformation space of convex co-compact hyperbolic metrics on a given manifold. Finally we will describe some open questions concerning the renormalized volume of convex co-compact hyperbolic manifolds having a given Riemann surface as asymptotic boundary.

Andrea SEPPI (CNRS Grenoble)

Maximal submanifolds in pseudo-hyperbolic space and their applications

The Asymptotic Plateau Problem is the problem of existence of submanifolds of vanishing mean curvature with prescribed boundary “at infinity”. It has been studied in the hyperbolic space, in the Anti-de Sitter space, and in several other contexts. In this talk, I will present the solution of the APP for complete spacelike maximal p -dimensional submanifolds in the pseudo-hyperbolic space of signature (p,q) . In the second part of the talk, I will discuss applications of this result in Teichmüller theory and for the study of Anosov representations. This is joint work with Graham Smith and Jérémy Toulisse.

Alice-Barbara TUMPACH (Lille/Pauli Institut Vienne)

The Universal Teichmueller space, the Siegel disc and the restricted Grassmannian

We explain the relation between the universal Teichmueller space and the following infinite-dimensional coadjoint orbits:

- the Kaehler coadjoint orbit $\text{Diff}(S^1)/\text{PSU}(1,1)$ of the Virasoro group (i.e. of the central extension of the group of diffeomorphisms of the circle)
- the Siegel disc and the restricted Siegel disc as coadjoint orbit of the infinite-dimensional symplectic group and its restricted version;
- the restricted Grassmannian as a kaehler coadjoint orbit of the restricted unitary group.

The universal Teichmueller space is closely related to quasi-symmetric homeomorphisms of the circle and their quasi-conformal extensions. It injects into the Siegel disc via the period mapping, and its connected component (for the Hilbert manifold structure introduced by Takhtajan and Teo) injects into the restricted Siegel disc, which itself injects holomorphically into the restricted Grassmannian. Here the Siegel disc is an infinite-dimensional generalization of the hyperbolic space and can be understood as a space of polarizations or complex structures on a real Hilbert space endowed with a strong symplectic form. The theory of the universal Teichmueller space is also linked to applications in pattern recognition and shape analysis via the fingerprint map. Some pathologies arising in the context of infinite-dimensional differential geometry will be mentioned, like the vanishing geodesic distance phenomenon that one encounters for example on the Virasoro group endowed with the right-invariant L^2 -metric and whose geodesic equation is given by the Korteweg-de Vries (KdV) equation, or the existence of queer Poisson brackets, i.e. Poisson brackets that do not act only on the first jet of functions. If time permits, the Poisson geometry of the restricted Grassmannian and its relation to the KdV equation will be briefly exposed.

Yilin WANG (IHES)

Circle homeomorphisms with square summable diamond shears

The shear coordinate is a countable coordinate system to describe increasing self-maps of the unit circle, which is furthermore invariant under modular transformations. Characterizations of circle homeomorphism, quasimetric homeomorphisms were obtained by D. Šarić. We are interested in characterizing Weil-Petersson circle homeomorphisms using shears. This class of homeomorphisms arises from the study of Kähler geometry on the universal Teichmüller space and connects various distant fields that will be mentioned briefly.

For this, we introduce diamond shear which is the minimal combination of shears producing WP homeomorphisms. Diamond shears are closely related to log-Lambda length introduced by R. Penner. We obtain sharp results comparing the class of circle homeomorphisms with square summable diamond shears with the Weil-Petersson class and Hölder classes. We also express the Weil-Petersson metric tensor and symplectic form in terms of infinitesimal shears and diamond shears.

This talk is based on joint work with Dragomir Šarić and Catherine Wolfram. See <https://arxiv.org/abs/2211.11497>.
